Date: / / Page no:

AME: MEET KATLANA

b: MATHS

Assignment: 1

Roll No: 17

Assignment-1

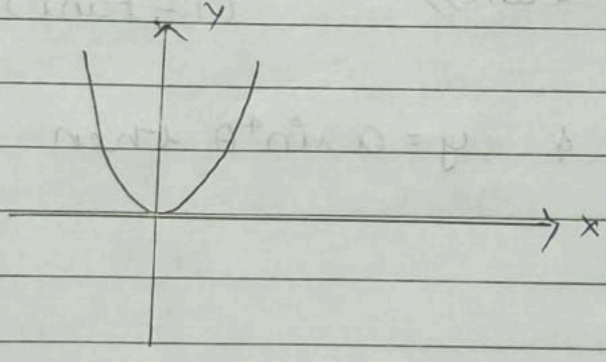
Q1 Explain the graphical representation of a functions by means of an example?

ans

Graphical presentation of a function refers to representing a mathematical function as a graph, typically on coordinate plane, where the x-axis (horizontal) represents the input values (domain), and the y-axis (vertical) represents the output value (ranges)

Example:-

Consider function $f(x) = x^2$.



Q2 Differentiate $\frac{(1+\tan x)}{(1-\tan x)}$ with respect to x?

ans $y = \frac{(1+\tan x)}{(1-\tan x)}$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{(1+\tan x)}{(1-\tan x)}\right)$$

$$= \frac{(1-\tan x)\left(\frac{d}{dx}(1+\tan x)\right) - (1+\tan x)\frac{d}{dx}(1-\tan x)}{(1-\tan x)^2}$$

$$\left\{\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{d}{dx}u - u\frac{d}{dx}v}{v^2}\right\}$$

$$= \frac{(1-\tan x)(\sec^2 x) - (1+\tan x)(-\sec^2 x)}{(1-\tan x)^2}$$

$$= \frac{\sec^2 x\,(1-\cancel{\tan x}+1+\cancel{\tan x})}{(1-\tan x)^2}$$

$$= \frac{2\sec^2 x}{(1-\tan x)^2}$$

Hence, $\frac{d}{dx}\left(\frac{(1+\tan x)}{(1-\tan x)}\right)$ is $\frac{2\sec^2 x}{(1-\tan x)^2}$

Q3 If $x = a\cos^4\theta$ & $y = a\sin^4\theta$ then find $\frac{dy}{dx}$?

Sol:- $x = a\cos^4\theta$

differentiate x with respect to θ.

$$\frac{dx}{d\theta} = a \cdot 4\cos^3\theta \cdot (-\sin\theta).$$

$$\frac{dx}{d\theta} = -4a\cos^3\theta\sin\theta. \longrightarrow (1)$$

$y = a \sin^4\theta$

differentiate y with respect to θ.

$\frac{dy}{d\theta} = a \cdot 4 \sin^3\theta \cdot \cos\theta$

$\frac{dy}{d\theta} = 4a \sin^3\theta \cos\theta$ — ②

divide eq. ② by eq. ①.

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cancel{4a} \sin^{\cancel{3}\,2}\theta \,\cancel{\cos\theta}}{-\cancel{4a} \cos^{\cancel{3}\,2}\theta \,\cancel{\sin\theta}}$

$= -\tan^2\theta$

Hence, $\frac{dy}{dx} = -\tan^2\theta$.

Q4 verify mean value theorem for $f(x) = x^2 - 4x - 3$ in the interval $[a, b]$ where $a = 1$ & $b = 4$.

sol: Given, function $f(x) = x^2 - 4x - 3$ is continuous in $[1, 4]$ & differentiable in $(1, 4)$.

then,

$f'(x) = 2x - 4$

$f'(c) = 2c - 4$

$f(a) = 1^2 - 4 \times 1 - 3 = 1 - 4 - 3 = -6 \quad \{a = 1\}$

$f(b) = 4^2 - 4 \times 4 - 3 = 16 - 16 - 3 = -3 \quad \{b = 4\}$

according to lagrange's mean value theorem,

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

$$2c-4 = \frac{-3-(-6)}{4-1}$$

$$2c-4 = \frac{-3+6}{3}$$

$$2c-4 = 3$$

$$\boxed{c = 5/2}$$

$c = 5/2 \in (1,4).$

Hence,
mean value theorem is verified.

Q5 Verify Rolle's theorem for the function $y = x^2+2$ $a=-2$ $b=2$.

sol: Given,
$f(x) = x^2+2$ is continuous in $[-2,2]$
& differential in $(-2,2)$
then,

$f(c) = c^2+2$ $\qquad f(a) = (-2)^2+2 = 6$

$f'(c) = 2c$ $\qquad f(b) = (2)^2+2 = 6$

acc. to rolles theorem,

$f'(c) = 0$ $\qquad \boxed{f(b) = f(a)}$

$2c = 0$

$\boxed{c = 0}$ $\qquad c = 0 \in (-2,2)$

Hence, rolles theorem verified.